F Theory Orientifolds, M Theory Orientifolds, and Twisted Strings

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Abstract

Orientifolds of the type IIB superstring that descend from F theory and M theory orbifolds are studied perturbatively. One finds strong evidence that a previously ignored twisted open string is required in these models. An attempt is made to interpret the J type torsion in F theory where one finds a realization of Gimon and Johnson's models which does not require these twisted strings. S-duality also provides evidence for these strings.

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1. Introduction

During the last year the recognition of Dirichlet(D)-branes [1] as the sought after Ramond-Ramond solitons in various weak-strong coupling dualities of string theory has renewed interest in deriving type I string theories as orientifolds of the type IIB theories [2-10]. Six-dimensional string theories, in particular, have received much attention partly due to a couple of factors. The massless spectrum of chiral theories in six dimensions is highly constrained by anomaly cancellation [11], and infrared divergences at singular points in the moduli space of these theories have been interpreted to signalize the appearance of fascinating but not well understood tensionless strings [12-16].

Studying M and F theories has enlarged the number and furthered the understanding of string dualities. The moduli space of heterotic string compactification to six dimensions has been described geometrically in F theory [17][18][19][20] and more intuitively in M theory [21]. This paper will continue the investigation of [22] into orientifolds that correspond to M and F theory models. In F theory we will study $\mathbf{Z_2} \times \mathbf{Z_n}$ and $\mathbf{Z_4}$ orbifolds of $\mathbf{T^6}$. Notations will be as in [22]. The $\mathbf{Z_2}$ will act on the elliptic fiber and correspond to $\Omega(-1)^{F_L} R_3$ in the IIB theory. The $\mathbf{Z_2} \times \mathbf{Z_2}$ case was previously discussed in [22]. For n even we will add discrete torsion and obtain some other models. It is important to emphasize that in Polchinski's notation [23] Ω rather than ΩJ should be used here. The details have not been worked out rigorously, but it appears that models sharing the same moduli space as that of Gimon and Johnson's models [8] can be achieved in F theory through a J type twist. There are other possible orbifolds involving $\mathbf{Z_2}$ shifts which will not be discussed here.[1] The irreducible anomaly of the massless spectrum will be exactly zero in the F theory models. However, we will discover in IIB string theory that we must introduce a previously unnoticed twisted open string to obtain agreement of the IIB and F theory spectrums and to cancel the IIB anomalies. We will show that the IIB tadpole anomaly is proportional to the anomaly of these twisted strings. Interestingly enough, one of the models with discrete torsion will turn out to be the ubiquitous model of [24][7][17].

Next we will find more evidence for these twisted strings by studying orientifolds of M theory on $\mathbf{T^5}/\mathbf{Z_n}$. The $\mathbf{Z_2} \times \mathbf{Z_n}$ models correspond to the heterotic string on $K3$ orbifolds or standard type I compactifications (1 tensor multiplet) on $K3$ [25][26][21] and will not be discussed here. We will show that the $\mathbf{T^5}/\mathbf{Z_4}$ and $\mathbf{T^5}/\mathbf{Z_6}$ orbifolds are related by T-duality to orientifolds of IIB. Naively, these orientifolds appeared to be the $\mathbf{Z_4}$ and $\mathbf{Z_6}$

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[1] A. Zaffaroni has considered these models.

B models of Gimon and Johnson [8] which did not seem to agree with M theory. The $SL(2, \mathbf{Z})$ (S)-duality of IIB was expected to interchange $(-1)^{F_L}$ and Ω [27][28]. However, the massless spectrum of IIB on the orbifold $\mathbf{T^4}/(-1)^{F_L}\mathbf{Z_4}$ did not match that of Gimon and Johnson on $\mathbf{T^4}/\Omega\mathbf{Z_4}$.[2] We will provide some evidence that the resolution of these puzzles is that the above Ω of Gimon and Johnson is really $\tilde{\Omega} = \Omega J$ as discussed in [26]. Replacing ΩJ by Ω will cause twisted strings to appear in the spectrum and resolve the S-duality puzzle. We will also find agreement with the M theory version of these models resolving the T-duality puzzle.

We will give two pieces of evidence for these twisted strings. The Klein Bottle contribution of the $\mathbf{Z_4}$ orientifold will equal the partition function of S-dual sectors corresponding to the $(-1)^{F_L}\mathbf{Z_4}$ theory when the modulus of the torus is purely imaginary and equals that of the Klein bottle. However, there is one subtlety here that only affects the massive spectrum:$(-1)^{F_L}$ should be replaced by $(-1)^{F_L}\Omega_b$ to get full agreement where Ω_b exchanges only left and right-moving, space-time bosonic oscillators. Given that the $(-1)^{F_L}\mathbf{Z_4}$ orbifold requires twisted closed strings to be a consistent, modular invariant theory we suspect that the $\Omega\mathbf{Z_4}$ theory will require a similar contribution. There is again a subtlety in this argument which leads to the second piece of evidence for these strings. It is possible to calculate a perturbative open string twisted by $\Omega J\mathbf{Z_4}$. This string is analogous to the left-moving sector of the $(-1)^{F_L}\mathbf{Z_4}$ twisted string. Applying Ω to this string gives the analogous right-moving sector. This string has unusual boundary conditions that make sense for a $\mathbf{Z_4}$ orbifold. However, twisting by $\Omega\mathbf{Z_4}$ gives a string without oscillator modes in the compactified directions violating supersymmetry since the Ramond(R) and Neveu-Schwarz(NS) sectors will not match. Since the twisted strings should exist in the Ω but not ΩJ theory, one suspects that Ω transforms into ΩJ under the "modular" (duality) transformation of the orientifold theory and that J projects out the unwanted Ω twisted strings in the ΩJ theory. Similar considerations will apply to the $\mathbf{Z_6}$ theory. We do not know whether these strings have a D-brane interpretation, perhaps by putting enough of them together to get a $\mathbf{Z_2}$ twisted object. Many of these questions will not be resolved in this paper.

This paper is organized as follows. Section two will discuss the F theory models first from the F theory perspective and then the IIB perturbative point of view. Section three will consider the M theory models showing first the S and T-duality relations and then providing further evidence for the twisted strings. Section four will discuss the results.

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[2] A. Zaffaroni noticed this discrepancy.

2. Orientifolds from F Theory

2.1. F theory orbifolds

Results from F theory orbifolds are summarized in the following table. The notations are as follows:$-$ is the $\mathbf{Z_2}$ generator, i is the $\mathbf{Z_4}$ generator, ω the $\mathbf{Z_6}$ generator, and ω^2 the $\mathbf{Z_3}$ generator. The action of a group element on $\mathbf{T^6}$ is given and followed by the Hodge numbers, h_{11} and h_{21}, for that sector. As in [22] the coordinates on $\mathbf{T^6}$ are z_3, z_4, and z_5. Tensors are abbreviated as T, vectors as V, hypermultiplets as H, discrete torsion as D.T., J will indicate another type of discrete torsion, and s will stand for sector. The action of group elements on the z_n is listed in the appropriate column.

Table 1:

F Theory Orientifolds

Model	(h_{11},h_{21})	T	V	H	z_5	z_3	z_4	$(h_{11},h_{21})^s$	T^s	V^s	H^s
$\mathbf{Z_2} \times \mathbf{Z_2}$	$(51,3)$	17	$SO(8)^8$	4	$+$	$+$	$+$	$(3,3)$	1	0	4
					$-$	$-$	$+$	$(16,0)$	0	$SO(8)^4$	0
					$-$	$+$	$-$	$(16,0)$	0	$SO(8)^4$	0
					$+$	$-$	$-$	$(16,0)$	16	0	0
$\mathbf{Z_4}$	$(31,7)$	13	$SO(8)^4$	8	$+$	$+$	$+$	$(5,1)$	3	0	2
					$+$	$-$	$-$	$(10,6)$	10	0	6
					$-$	i	i	$(16,0)$	0	$SO(8)^4$	0
					$-$	$-i$	$-i$	$(0,0)$			
$\mathbf{Z_2} \times \mathbf{Z_3}$	$(35,11)$	13	$SO(8)^5$	$SO(8)+8$	$+$	$+$	$+$	$(3,1)$	1	0	2
					$+$	ω^2	ω^4	$(6,3)$	6	0	3
					$+$	ω^4	ω^2	$(6,3)$	6	0	3
					$-$	$-$	$+$	$(8,4)$	0	$SO(8)^2$	$SO(8)$
					$-$	ω	ω^2	$(12,0)$	0	$SO(8)^3$	0
					$-$	ω^5	ω^4	$(0,0)$			

(table cont'd)

(table 1 cont'd)

$\mathbf{Z_2 \times Z_4}$	$(61,1)$	19	$SO(8)^{10}$	2	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(12,0)$	0	$SO(8)^3$	0
					$-$	$+$	$-$	$(12,0)$	0	$SO(8)^3$	0
					$+$	$-$	$-$	$(10,0)$	10	0	0
					$+$	i	$-i$	$(4,0)$	4	0	0
					$+$	$-i$	i	$(4,0)$	4	0	0
					$-$	i	i	$(16,0)$	0	$SO(8)^4$	0
					$-$	$-i$	$-i$	$(0,0)$			
$\mathbf{Z_2 \times Z_6}$	$(51,3)$	17	$SO(8)^8$	4	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(8,0)$	0	$SO(8)^2$	0
					$-$	$+$	$-$	$(8,0)$	0	$SO(8)^2$	0
					$+$	$-$	$-$	$(6,0)$	6	0	0
					$+$	ω	ω^5	$(1,0)$	1	0	0
					$+$	ω^2	ω^4	$(4,1)$	4	0	1
					$+$	ω^4	ω^2	$(4,1)$	4	0	1
					$+$	ω^5	ω	$(1,0)$	1	0	0
					$-$	ω	ω^2	$(8,0)$	0	$SO(8)^2$	0
					$-$	ω^2	ω	$(8,0)$	0	$SO(8)^2$	0
					$-$	ω^4	ω^5	$(0,0)$			
					$-$	ω^5	ω^4	$(0,0)$			
$\mathbf{Z_2 \times Z_2}$,D.T.	$(3,51)$	1	0	$SO(8)^8 + 20$	$+$	$+$	$+$	$(3,3)$	1	0	4
					$-$	$-$	$+$	$(0,16)$	0	0	$SO(8)^4$
					$-$	$+$	$-$	$(0,16)$	0	0	$SO(8)^4$
					$+$	$-$	$-$	$(0,16)$	0	0	16

(table cont'd)

(table 1 cont'd)

$\mathbf{Z_2} \times \mathbf{Z_4}$,D.T.	$(21,9)$	11	$SO(8)^2$	10	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(4,0)$	0	$SO(8)$	0
					$-$	$+$	$-$	$(4,0)$	0	$SO(8)$	0
					$+$	$-$	$-$	$(10,0)$	10	0	0
					$+$	i	$-i$	$(0,4)$	0	0	4
					$+$	$-i$	i	$(0,4)$	0	0	4
					$-$	i	i	$(0,0)$			
					$-$	$-i$	$-i$	$(0,0)$			
$\mathbf{Z_2} \times \mathbf{Z_6}$,D.T.	$(19,19)$	9	$SO(8)^2$	$SO(8)^2 + 12$	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(0,4)$	0	0	$SO(8)$
					$-$	$+$	$-$	$(0,4)$	0	0	$SO(8)$
					$+$	$-$	$-$	$(0,6)$	0	0	6
					$+$	ω	ω^5	$(0,1)$	0	0	1
					$+$	ω^2	ω^4	$(4,1)$	4	0	1
					$+$	ω^4	ω^2	$(4,1)$	4	0	1
					$+$	ω^5	ω	$(0,1)$	0	0	1
					$-$	ω	ω^2	$(4,0)$	0	$SO(8)$	0
					$-$	ω^2	ω	$(4,0)$	0	$SO(8)$	0
					$-$	ω^4	ω^5	$(0,0)$			
					$-$	ω^5	ω^4	$(0,0)$			
$\mathbf{Z_2} \times \mathbf{Z_3}, J$	$(20,14)$	10	$SO(8)^2$	$SO(8) + 11$	$+$	$+$	$+$	$(3,1)$	1	0	2
					$+$	ω^2	ω^4	$(3,6)$	3	0	6
					$+$	ω^4	ω^2	$(6,3)$	6	0	3
					$-$	$-$	$+$	$(8,4)$	0	$SO(8)^2$	$SO(8)$
					$-$	ω	ω^2	$(0,0)$			
					$-$	ω^5	ω^4	$(0,0)$			

(table cont'd)

$\mathbf{Z_4}, J$	$(11,11)$	9	0	12	$+$	$+$	$+$	$(5,1)$	3	0	2
					$+$	$-$	$-$	$(6,10)$	6	0	10
					$-$	i	i	$(0,0)$			
					$-$	$-i$	$-i$	$(0,0)$			
$\mathbf{Z_2} \times \mathbf{Z_4}, J$	$(7,31)$	5	0	$16 + SO(8)^4$	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(0,12)$	0	0	$SO(8)^3$
					$-$	$+$	$-$	$(0,4)$	0	0	$SO(8)$
					$+$	$-$	$-$	$(0,10)$	0	0	10
					$+$	i	$-i$	$(0,4)$	0	0	4
					$+$	$-i$	i	$(4,0)$	4	0	0
					$-$	i	i	$(0,0)$			
					$-$	$-i$	$-i$	$(0,0)$			
$\mathbf{Z_2} \times \mathbf{Z_6}, J$	$(9,21)$	7	0	$14 + SO(8)^3$	$+$	$+$	$+$	$(3,1)$	1	0	2
					$-$	$-$	$+$	$(0,4)$	0	0	$SO(8)$
					$-$	$+$	$-$	$(0,4)$	0	0	$SO(8)$
					$+$	$-$	$-$	$(6,0)$	6	0	0
					$+$	ω	ω^5	$(0,1)$	0	0	1
					$+$	ω^2	ω^4	$(0,5)$	0	0	5
					$+$	ω^4	ω^2	$(0,5)$	0	0	5
					$+$	ω^5	ω	$(0,1)$	0	0	1
					$-$	ω	ω^2	$(0,0)$			
					$-$	ω^2	ω	$(0,0)$			
					$-$	ω^4	ω^5	$(0,0)$			
					$-$	ω^5	ω^4	$(0,0)$			

In the above table a few points need further clarification. We have assumed that the gauge group or global symmetry in these models is always $SO(8)$. The justification is that Morrison and Vafa [19] have stated and Sen [29] has shown in detail that a $\mathbf{Z_2}$ orbifold singularity on the fiber corresponds to a D_4 singularity yielding an $SO(8)$ group. Discrete

torsion has been described by [30]:In a $\mathbf{Z_2} \times \mathbf{Z_{2n}}$ model the discrete torsion lies in $\mathbf{Z_2}$. Let a be the generator of $\mathbf{Z_2}$ and b the generator of $\mathbf{Z_{2n}}$. A generic group element $a^{m_1} b^{n_1}$ in the sector twisted by $a^{m_2} b^{n_2}$ receives an extra factor $(-1)^{m_1 n_2 - m_2 n_1}$ when one projects in that sector by summing over the action of group elements.

The J type of torsion appears to be more complicated but seems worth understanding since some interesting models are realized by it. J can be described as follows. For the $\mathbf{Z_2} \times \mathbf{Z_{2n}}$ cases, $\mathbf{Z_{2n}}$ twists, t_2, are assigned a value in $\mathbf{Z_2}$. Using the same notation as for the discrete torsion case, the value is 0 if $n_2 = 0$, mod 3 and 1 otherwise. The values of $p_1 \equiv m_1$ and $t_1 \equiv m_2$ already lie in $\mathbf{Z_2}$. For the $\mathbf{Z_2} \times \mathbf{Z_3}$ case, $t_2 = n_2$, mod 2, and for the $\mathbf{Z_4}$ case $t_2 = 1$ for twisted sectors and 0 for the untwisted sector. One defines $p_2 = n_1$, mod 2 for the $\mathbf{Z_2} \times \mathbf{Z_6}$ case, $p_2 = 0$ for the $\mathbf{Z_2} \times \mathbf{Z_3}$ case, and $p_2 = n_1 \in \mathbf{Z_4}$ for the other two cases. Then, J acts by giving an extra $\mathbf{Z_2}$ or $\mathbf{Z_4}$ factor to projectors in the various twisted sectors and with a $\mathbf{Z_2}$ action on z_3. This can be summarized in the following table.

Table 2:

J Torsion

Model	J	Action on z_3
$\mathbf{Z_2} \times \mathbf{Z_3}$	$e^{\pi i t_2 p_1}$	1
$\mathbf{Z_4}$	$e^{\pi i t_2 p_2}$	1
$\mathbf{Z_2} \times \mathbf{Z_4}$	$e^{\pi i t_2 p_1} e^{\frac{\pi i}{2} t_1 p_2}$	1
$\mathbf{Z_2} \times \mathbf{Z_6}$	$e^{\pi i t_2 p_1} e^{\pi i t_1 p_2}$	$e^{\pi i t_2 p_1}$

Several points are worth mentioning before we discuss the above models as IIB orientifolds. The irreducible anomalies vanish for all of these models, and most can be interpreted as Voisin-Borcea models [31][32]. It is interesting to note that a continuation to negative $k = (r - a)/2$ of the Voisin-Borcea classification elucidated by [19] applies to the $(3, 51)$ if we choose $r = 2$ and $a = 4$. A similar remark can be made for the other J models. The $\mathbf{Z_2} \times \mathbf{Z_6}$ model with discrete torsion provides another F theory realization of the Sen model [24][33], and we will encounter this same model as M theory on $\mathbf{T^5}/\mathbf{Z_6}$. Finally, we are able to realize models in F theory sharing the moduli space of the Gimon and Johnson models by applying the J type torsion. The $\mathbf{Z_2} \times \mathbf{Z_2}$ case is the regular discrete torsion yielding the $(3, 51)$. One obtains a couple of mirror orbifolds from this construction. Notice from Table 1 that the twisted sectors that will correspond to the new type of twisted string are absent here, giving evidence that it was correct for Gimon and

Johnson to ignore these strings in their models. Whether there is a more systematic or mathematical description of J in the F theory context is not known to me. Also, whether tensionless strings play a role in these models remains to be seen.

2.2. IIB orientifolds

Closed string results

The table below shows the discrete orientifold groups of the IIB models corresponding to the F theory models.

Table 3:

IIB orientifolds

Model	Orientifold Generators
$\mathbf{Z_2} \times \mathbf{Z_2}$	$(\Omega(-1)^{F_L} R_3, \Omega(-1)^{F_R} R_4)$
$\mathbf{Z_4}$	$(\Omega(-1)^{F_L} R_3 \alpha)$
$\mathbf{Z_2} \times \mathbf{Z_3}$	$(\Omega(-1)^{F_L} R_3, \omega^2)$
$\mathbf{Z_2} \times \mathbf{Z_4}$	$(\Omega(-1)^{F_L} R_3, \alpha)$
$\mathbf{Z_2} \times \mathbf{Z_6}$	$(\Omega(-1)^{F_L} R_3, \omega)$

Here, $(-1)^{F_L} = e^{2\pi i s_1^L}$, $R_3 = e^{\pi i s_3}$, $R_4 = e^{-\pi i s_4}$, $\alpha = e^{\frac{\pi i (s_3 - s_4)}{2}}$, and $\omega = e^{\frac{i\pi(s_3 - s_4)}{3}}$. Again, the notations follow [22] with $s_3 = s_3^L + s_3^R$ and s the spin. Discrete torsion in various twisted sectors is exactly as in the F theory description. We will not discuss the J models which should correspond to the Gimon and Johnson models. The first thing to notice about these models is that all cases except the $\mathbf{Z_2} \times \mathbf{Z_3}$ and $\mathbf{Z_4}$ cases contain a $\mathbf{Z_2} \times \mathbf{Z_2}$ subgroup. Thus, the results of [22] apply, and there will be 32 each of the two kinds of seven-branes with matter projected out in the $7 - 7'$ sector. The matrices acting on Chan-Paton factors will be exactly as in [22] for the $\mathbf{Z_2} \times \mathbf{Z_2}$ sectors. Thus, what remains is to determine the other twisted sectors. Before doing this, let us determine the closed string spectrum of the models. The following tables show the general case and then specific results from our models. Let $g = e^{\frac{2\pi i}{n}(s_3 - s_4)}$ be the $\mathbf{Z_n}$ generator.

Table 4

Left-Moving States

Sector	State	$(-1)^{F_L} R_3$	g^m
untwisted			
NS :	$\psi^{\mu}_{-1/2}\lvert 0>$	1	1
	$\psi^{z_3}_{-1/2}\lvert 0>$	-1	$e^{\frac{2\pi i m}{n}}$
	$\psi^{\bar{z}_3}_{-1/2}\lvert 0>$	-1	$e^{\frac{-2\pi i m}{n}}$
	$\psi^{z_4}_{-1/2}\lvert 0>$	1	$e^{\frac{-2\pi i m}{n}}$
	$\psi^{\bar{z}_4}_{-1/2}\lvert 0>$	1	$e^{\frac{2\pi i m}{n}}$
R :	$s^L_1 = s^L_2, s^L_3 = s^L_4 = \frac{1}{2}$	$-i$	1
	$s^L_1 = s^L_2, s^L_3 = s^L_4 = \frac{-1}{2}$	i	1
	$s^L_1 = -s^L_2, s^L_3 = -s^L_4 = \frac{1}{2}$	$-i$	$e^{\frac{2\pi i m}{n}}$
	$s^L_1 = -s^L_2, s^L_3 = -s^L_4 = \frac{-1}{2}$	i	$e^{\frac{-2\pi i m}{n}}$
twisted by $g^p \neq 1/2$			
NS :	$\psi^{z_3}_{p/n-1/2}\lvert 0>$	$e^{\pi i (1-\frac{p}{n})}$	$e^{2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
	$\psi^{\bar{z}_4}_{p/n-1/2}\lvert 0>$	$e^{-\pi i \frac{p}{n}}$	$e^{2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
R :	$s^L_1 = -s^L_2$	$e^{\pi i (\frac{3}{2}-\frac{p}{n})}$	$e^{2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
twisted by $1/2$, n even			
NS :	$s^L_3 = s^L_4 = \frac{1}{2}$	i	1
	$s^L_3 = s^L_4 = \frac{-1}{2}$	$-i$	1
R :	$s^L_1 = -s^L_2$	-1	1

Sector	State	$(-1)^{F_L}R_3$	g^m
untwisted			
NS :	$\psi^{\mu}_{-1/2}\|0>$	1	1
	$\psi^{z_3}_{-1/2}\|0>$	-1	$e^{\frac{2\pi i m}{n}}$
	$\psi^{\bar{z}_3}_{-1/2}\|0>$	-1	$e^{\frac{-2\pi i m}{n}}$
	$\psi^{z_4}_{-1/2}\|0>$	1	$e^{\frac{-2\pi i m}{n}}$
	$\psi^{\bar{z}_4}_{-1/2}\|0>$	1	$e^{\frac{2\pi i m}{n}}$
R :	$s^R_1 = s^R_2, s^R_3 = s^R_4 = \frac{1}{2}$	i	1
	$s^R_1 = s^R_2, s^R_3 = s^R_4 = \frac{-1}{2}$	$-i$	1
	$s^R_1 = -s^R_2, s^R_3 = -s^R_4 = \frac{1}{2}$	i	$e^{\frac{2\pi i m}{n}}$
	$s^R_1 = -s^R_2, s^R_3 = -s^R_4 = \frac{-1}{2}$	$-i$	$e^{\frac{-2\pi i m}{n}}$
twisted by $g^p \neq 1/2$			
NS :	$\psi^{z_3}_{p/n-1/2}\|0>$	$e^{\pi i \frac{p}{n}}$	$e^{-2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
	$\psi^{z_4}_{p/n-1/2}\|0>$	$e^{-\pi i(1-\frac{p}{n})}$	$e^{-2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
R :	$s^R_1 = -s^R_2$	$e^{-\pi i(\frac{1}{2}-\frac{p}{n})}$	$e^{-2\pi i \frac{m}{n}(1-\frac{2p}{n})}$
twisted by $1/2$, n even			
NS :	$s^R_3 = s^R_4 = \frac{1}{2}$	i	1
	$s^R_3 = s^R_4 = \frac{-1}{2}$	$-i$	1
R :	$s^R_1 = -s^R_2$	1	1

Because we are using Ω not ΩJ, some of the phases are different from that case. As usual, Ω gives a positive sign for symmetric left and right Neveu-Schwarz–Neveu-Schwarz states but a negative sign for symmetric left and right Ramond-Ramond states. In every model considered the untwisted sector gives the $N = 1$ supergravity multiplet. In the twisted sectors, the action of g on the fixed points must be taken into account, and similarly the discrete torsion projections should be remembered.

Closed string sectors of IIB models

Model	Twist	T	H
$\mathbf{Z_2 \times Z_2}$	1	1	4
	1/2	16	0
$\mathbf{Z_4}$	1	3	2
	1/2	10	6
$\mathbf{Z_2 \times Z_3}$	1	1	2
	1/3	6	3
	2/3	6	3
$\mathbf{Z_2 \times Z_4}$	1	1	2
	1/2	10	0
	1/4	4	0
	3/4	4	0
$\mathbf{Z_2 \times Z_6}$	1	1	2
	1/2	6	0
	1/3	4	1
	2/3	4	1
	1/6	1	0
	5/6	1	0
$\mathbf{Z_2 \times Z_2}$,D.T.	1	1	4
	1/2	0	16

Model	Twist	T	H
$\mathbf{Z_2 \times Z_4}$,D.T.	1	1	2
	1/2	10	0
	1/4	0	4
	3/4	0	4
$\mathbf{Z_2 \times Z_6}$,D.T.	1	1	2
	1/2	0	6
	1/3	4	1
	2/3	4	1
	1/6	0	1
	5/6	0	1

These results agree sector by sector with those
found in the last section from F theory.

Tadpoles

The general formalism of Gimon and Polchinski [6] applies to the calculation of tadpole anomalies in the above theories. It is a bit too tedious to show all of the theta functions that enter these calculations, but there are several points to keep in mind. The projector $\Omega(-1)^{F_L}R_3$ as compared to Ω gives an extra minus sign on the closed vacuums twisted by $\frac{1}{2}$, changing the sign of some Klein bottle and Mobius strip terms. The untwisted Klein bottle gets no extra factor when one traces the g action on the four-torus because the sum over fixed points cancels the continuous sum(integral). However, the $\frac{1}{2}$ twisted terms in the Klein bottle generally do get a fixed point factor. Also, the sum over fixed points of R_3 and R_4 must be accompanied by appropriate phases for the action of g on the fixed points. In fact, we will diagonalize the action on combinations of fixed points.

Tadpoles twisted by $\frac{1}{2}$ for the $\mathbf{Z_2} \times \mathbf{Z_2}$ case with discrete torsion get an overall minus with respect to the $\mathbf{Z_2} \times \mathbf{Z_2}$ without discrete torsion. The matrices operating on Chan-Paton factors are the same as for that case, but the extra minus projects out the $SO(8)^8$ vectors and keeps the global $SO(8)^8$ hypermultiplets. The result agrees with F theory. Notice that if there were not a global $SO(8)^8$ symmetry, this model would be the same as the $(3, 243)$ which is also the Gimon and Polchinski model at a generic point in the moduli space. There appears to be no way to deform this orientifold because moving branes away from the fixed points would ruin the $SO(8)$ symmetry and cause the theory to be anomalous.

Tadpoles for the other models are listed as follows. All are proportional to $v_6 \int_0^\infty dl$ where $v_6 = V_6/(4\pi\alpha')^3$ and V_6 is the noncompact volume.

$$\mathbf{Z_4} : \frac{1}{16}\sum_{I,I'}(\beta_I\beta_{I'}16)^2 \tag{2.1}$$

$$\mathbf{Z_2} \times \mathbf{Z_4} : \begin{array}{l} \dfrac{1}{32}\sum_{I,I'}(\mathrm{Tr}\gamma^I_{1/2} - \mathrm{Tr}\gamma^{I'}_{1/2} - \beta_I\beta_{I'}16)^2 \\[2mm] (\dfrac{1}{32}\sum_{I,I'}\alpha_I\alpha'_{I'}(\mathrm{Tr}\gamma^I_{1/4} - \mathrm{Tr}\gamma^{I'}_{1/4})^2) + (1/4 \to 3/4) \end{array} \tag{2.2}$$

$$\mathbf{Z_2} \times \mathbf{Z_3} : (\frac{3}{24}\sum_I \omega_I(\mathrm{Tr}\gamma^I_{1/3} - \beta'_I 8)^2) + (1/3 \to 2/3) \tag{2.3}$$

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$$\mathbf{Z_2} \times \mathbf{Z_6}: \quad \begin{aligned} &(\frac{3}{48}\sum_{I,I'}\omega'_I\omega''_{I'}(\mathrm{Tr}\gamma^I_{1/3}-\mathrm{Tr}\gamma^{I'}_{1/3}-\beta'_I 8-\beta'_{I'} 8)^2) + (1/3\to 2/3) \\ &(\frac{1}{48}\sum_{I,I'}\omega'^2_I\omega''^2_{I'}(\mathrm{Tr}\gamma^I_{1/6}-\mathrm{Tr}\gamma^{I'}_{1/6})^2) + (1/6\to 5/6) \end{aligned} \tag{2.4}$$

$$\mathbf{Z_2} \times \mathbf{Z_4}, D.T.: \quad \begin{aligned} &\frac{1}{32}\sum_{I,I'}(\mathrm{Tr}\gamma^I_{1/2}-\mathrm{Tr}\gamma^{I'}_{1/2}-\beta_I\beta_{I'}16)^2 \\ &(-\frac{1}{32}\sum_{I,I'}\alpha_I\alpha'_{I'}(\mathrm{Tr}\gamma^I_{1/4}-\mathrm{Tr}\gamma^{I'}_{1/4})^2) + (1/4\to 3/4) \end{aligned} \tag{2.5}$$

$$\mathbf{Z_2} \times \mathbf{Z_6}, D.T.: \quad \begin{aligned} &(\frac{3}{48}\sum_{I,I'}\omega'_I\omega'_{I'}(\mathrm{Tr}\gamma^I_{1/3}-\mathrm{Tr}\gamma^{I'}_{1/3})^2) + (1/3\to 2/3) \\ &(-\frac{1}{48}\sum_{I,I'}\omega'^2_I\omega'^2_{I'}(\mathrm{Tr}\gamma^I_{1/6}-\mathrm{Tr}\gamma^{I'}_{1/6})^2) + (1/6\to 5/6) \end{aligned} \tag{2.6}$$

The notations used in the above equations are defined below. All blocks are eight dimensional in the case of vectors and 8×8 dimensional in the case of matrices. In the above I runs over the fixed points of R_3 and I' over those of R_4. In the solutions that follow, I have not been terribly concerned about proving their uniqueness but have chosen the phases to obtain the desired result that is consistent with space-time anomaly cancellation.

$$\alpha_I = (1, -1, 1, 1)$$
$$\alpha'_{I'} = (-1, 1, 1, 1)$$
$$\omega_I = (1, 1, \omega^2, \omega^4) \tag{2.7}$$
$$\omega'_I = \omega_I$$
$$\omega''_{I'} = (1, 1, \omega^4, \omega^2)$$

$$\beta_I = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\beta'_I = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

$$\gamma_{\Omega(-1)^{F_L} R_3} = \gamma^I_{1/2} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\gamma^I_{1/4} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \tag{2.8}$$

$$\gamma^I_{1/3} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \omega^2 & \\ & & & \omega^4 \end{pmatrix}$$

$$\gamma^I_{1/6} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \omega^4 & \\ & & & \omega^2 \end{pmatrix}$$

These matrices imply that D-branes yield the following matter content and left-over tadpole anomaly. The anomaly is multiplied by $v_6 \int_0^\infty dl$ in the following table and the expected extra vectors from F theory are listed in the last column.

Table 6:

Tadpole Anomalies and Extra Gauge Fields from F Theory

Model	V	H	Anomaly	Extra Vectors
$\mathbf{Z_2 \times Z_2}$	$SO(8)^8$	0	0	0
$\mathbf{Z_4}$	0	0	64	$SO(8)^4$
$\mathbf{Z_2 \times Z_3}$	$SO(8)^2$	$SO(8)$	48	$SO(8)^3$
$\mathbf{Z_2 \times Z_4}$	$SO(8)^6$	0	64	$SO(8)^4$
$\mathbf{Z_2 \times Z_6}$	$SO(8)^4$	0	64	$SO(8)^4$
$\mathbf{Z_2 \times Z_2}$, D.T.	0	$SO(8)^8$	0	0
$\mathbf{Z_2 \times Z_4}$, D.T.	0	$SO(8)^2$	0	0
$\mathbf{Z_2 \times Z_6}$, D.T.	0	$SO(8)^2$	32	$SO(8)^2$

The results are again in agreement with F theory as the left-over tadpole anomaly is proportional to the irreducible anomaly, to the number of $SO(8)$ vectors necessary to cancel this anomaly, and to the expected extra vectors from F theory. We, thus, assume that there are twisted open strings present in these theories and will provide other evidence for these objects in the M theory discussion.

3. Orientifolds from M Theory

3.1. T-duality, S-duality, and IIB orientifolds

In this section two M theory orbifolds, $\mathbf{T^5/Z_4}$ and $\mathbf{T^5/Z_6}$, will be studied. The $\mathbf{Z_4}$ generator is $(-, \alpha)$ and the $\mathbf{Z_6}$ generator $(-, \omega)$ where the minus acts on the fifth circle while α or ω is acting on the four-torus. We will first give an argument for why these models should be equivalent to the $(K3 \times \mathbf{S^1})/\mathbf{Z_2}$ first discussed by [24]. Writing $\mathbf{Z_4}$ ($\mathbf{Z_6}$) as an internal direct product $\mathbf{Z_4/Z_2 \times Z_2}$ ($\mathbf{Z_6/Z_3 \times Z_3}$), we see that the two models are equivalent to $(K3 \times \mathbf{S^1})/\mathbf{Z_2}$. This extra $\mathbf{Z_2}$ acts in both cases with a -1 on $\mathbf{S^1}$ and eight -1's on the 19 antiself-dual two-forms of $K3$. The action is not the same as in [24], for only some of the -1's come from exchanging part of the two E_8 lattices. Presumably, there is an $SO(19)$ rotation that relates the three $\mathbf{Z_2}$'s. In any case the spectrum of the three models using the argument of [24] is 9 tensors, 20 hypermultiplets, and a gauge group of

rank eight with hypermultiplets in the adjoint of this gauge group. This spectrum also corresponds to F theory on the $(19, 19)$.

Using the prescription of [34] to translate the models into IIA theory, we get the orbifolds $(-1)^{F_L}\alpha$ and $(-1)^{F_L}\omega$. The massless spectrum of IIA and IIB on these orbifolds is given in the following table.

Table 7:

Type II Strings on $(-1)^{F_L}\alpha$ and $(-1)^{F_L}\omega$

Model	Sector	T	V	H
IIA on $\mathbf{T^4}/(-1)^{F_L}\alpha$	1	1	2	2
	α^2	0	6	10
	$(-1)^{F_L}\alpha$	4	0	4
	$(-1)^{F_L}\alpha^3$	4	0	4
IIA on $\mathbf{T^4}/(-1)^{F_L}\omega$	1	1	0	2
	ω^2	0	4	5
	ω^4	0	4	5
	$(-1)^{F_L}\omega^3$	6	0	6
	$(-1)^{F_L}\omega$	1	0	1
	$(-1)^{F_L}\omega^5$	1	0	1
IIB on $\mathbf{T^4}/(-1)^{F_L}\alpha$	1	3	0	2
	α^2	6	0	10
	$(-1)^{F_L}\alpha$	0	4	4
	$(-1)^{F_L}\alpha^3$	0	4	4
IIB on $\mathbf{T^4}/(-1)^{F_L}\omega$	1	1	0	2
	ω^2	4	0	5
	ω^4	4	0	5
	$(-1)^{F_L}\omega^3$	0	6	6
	$(-1)^{F_L}\omega$	0	1	1
	$(-1)^{F_L}\omega^5$	0	1	1

Notice that if one excludes one tensor from the untwisted sectors, the IIB models are obtained from the IIA models by exchanging vectors and tensors. Because the two models are the same, this exchange is a symmetry (as long as the gauge group is abelian). This result is expected since the spectrum matches the $(19, 19)$ which is mirror symmetric. If we compactify M theory on a circle, we obtain the orientifolds of the IIA theory on $\mathbf{T^5/Z_4}$ and $\mathbf{T^5/Z_6}$. Applying T-duality on the fifth circle gives IIB on the orientifolds $(\mathbf{T^4}/\Omega\alpha) \times \mathbf{S^1}$ and $(\mathbf{T^4}/\Omega\omega) \times \mathbf{S^1}$. The expected spectrum of these models is clearly different from the results of Gimon and Johnson reduced to five dimensions.[3] The expected massless spectrum agrees with the $(-1)^{F_L}$ models as S-duality arguments would predict. However, the expected results require again twisted open strings which we discuss in the next section.

3.2. Twisted Strings

The first piece of evidence for twisted strings will come from comparing the Klein bottle term of the $\Omega\mathbf{Z_4}$ model to the corresponding sectors of the partition function of the $(-1)^{F_L}\mathbf{Z_4}$ model. Using the notations of [6] the Klein bottle amplitude in the loop formulation is

$$(1 - 1)\frac{v_6}{16} \int_0^\infty \frac{dt}{t^4} \left(8\frac{f_3^4(e^{-2\pi t})f_4^4(e^{-2\pi t})}{f_1^4(e^{-2\pi t})f_2^4(e^{-2\pi t})} - 8\frac{f_2^4(e^{-2\pi t})f_4^4(e^{-2\pi t})}{f_1^4(e^{-2\pi t})f_3^4(e^{-2\pi t})}\right). \tag{3.1}$$

By doing a duality transformation $t \to \frac{1}{t}$, one sees that the divergence vanishes so this would appear to be a consistent closed string theory as noted by [8].

Corresponding to the Klein bottle, we examine the following sectors $Z(t_\sigma, t_\tau)$ of the $(-1)^{F_L}\alpha$ toroidal partition function: $Z(1, (-1)^{F_L}\alpha) + Z(1, (-1)^{F_L}\alpha^3) + Z(\alpha^2, (-1)^{F_L}\alpha) + Z(\alpha^2, (-1)^{F_L}\alpha^3)$. The modulus of the torus is set to be $\tau = it$ and t_σ (t_τ) is the twist in the spatial(time) direction of the torus. This gives

$$
4\frac{\left(\Theta^2\binom{0}{0}\Theta\binom{0}{1/4}\Theta\binom{0}{0} - \Theta^2\binom{0}{1/2}\Theta\binom{0}{3/4}\Theta\binom{0}{-3/4}\right)^2 - \Theta^4\binom{1/2}{0}\Theta^2\binom{1/2}{1/4}\Theta^2\binom{1/2}{-1/4}}{\eta^{12}\Theta^2\binom{1/2}{1/4}\Theta^2\binom{1/2}{-1/4}}
$$
$$
-\frac{\left(\Theta^2\binom{0}{0}\Theta\binom{1/2}{1/4}\Theta\binom{1/2}{-1/4} - \Theta^2\binom{0}{1/2}\Theta\binom{1/2}{3/4}\Theta\binom{1/2}{-3/4}\right)^2 + \Theta^4\binom{1/2}{0}\Theta^2\binom{0}{1/4}\Theta^2\binom{0}{-1/4}}{\eta^{12}\Theta^2\binom{0}{1/4}\Theta^2\binom{0}{-1/4}}. \tag{3.2}
$$

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[3] These T-dual models were obtained in discussions with S. Mukhi.

Here the notations are from the appendix of [35] with $q = e^{-2\pi t}$. Using identities found in [8], the above reduces to

$$(1-1)(8\frac{f_3^4(e^{-2\pi t})f_4^4(e^{-2\pi t})}{\eta^8(e^{-2\pi t})f_2^4(e^{-2\pi t})} - 8\frac{f_2^4(e^{-2\pi t})f_4^4(e^{-2\pi t})}{\eta^8(e^{-2\pi t})f_3^4(e^{-2\pi t})}) = (1-1)8\frac{f_4^2}{\eta^8 f_2^4 f_3^4}. \qquad (3.3)$$

This matches the Klein bottle term if one replaces η^8 by f_1^4. Since S-duality mixes up the R-R and NS-NS sectors, the $(1-1)$ here does not correspond to NS-NS$-$R-R. The factor f_1^4 or η^8 represents the bosonic space-time oscillators and only affects the massive spectrum. Perhaps, this difference between the two theories reflects a renormalization and would be absent if perturbative and nonperturbative corrections were taken into account. The difference can be eliminated if it is sensible in a perturbative framework to define an operator Ω_b which exchanges left and right-moving, space-time bosonic oscillators. Then S-duality would exchange Ω and $(-1)^{F_L}\Omega_b$. In order for the $(-1)^{F_L}\alpha$ theory to be consistent and modular invariant, strings twisted by $(-1)^{F_L}\alpha$ are required. At the modulus $\tau_1 = 0$, the above sectors of the partition function are invariant under $\tau \to \tau + 1$ so the modular transformation that produces these strings corresponds to $t \to \frac{1}{t}$ as with the Klein bottle. Since there is no phase transition at $\tau_1 = 0$, even at $t = 0$ (The only dangerous term is the massless contribution which vanishes.), one would not expect these strings to disappear there. Thus, the $t \to \frac{1}{t}$ transformation should also produce these strings in the Ω theory. Since there is no space-time anomaly, their contribution to the Klein bottle vanishes.

The second piece of evidence for these strings is to provide a perturbative formulation of them. If we try to solve for a string twisted by $\Omega\alpha$, we obtain an open string stuck at the fixed point with no zero or oscillator modes on the four-torus. Thus, the vacuum energy of the Ramond sector is 0 and that of the Neveu-Schwarz sector is $-1/4$ so these sectors do not match and supersymmetry is violated. These strings must somehow be projected out of the spectrum. The hypothesis is that J is the required projection and that twisting with Ω corresponds to projecting with ΩJ and vice versa. In the F theory version of J, one might postulate that a transition to the smooth Calabi-Yau is impossible and that blowing up the fixed point might create a superpotential along the lines of [36][37]. On the other hand, if we try to solve the equation $Z_3(\sigma + \pi) = \Omega J\alpha Z_3(\sigma) = \Omega\alpha Z_3^c(\sigma)$, we do get a viable open string. Here, J is the operator that switches the twist from 1/4 to 3/4 and if Z_3 is twisted by 1/4, Z_3^c is twisted by 3/4. The solution for Z_3 is the following:

$$Z_3 = z_3^{fix} + \sum_n (\alpha^3_{-n-1/4} e^{-i(n+\frac{1}{4})2(\tau-\sigma)} + \tilde{\alpha}^3_{-n-3/4} e^{-i(n+\frac{3}{4})2(\tau+\sigma)}) \qquad (3.4)$$

and

$$Z_3^c = z_3^{fix} + \sum_n (i\alpha^3_{-n-1/4} e^{-i(n+\frac{1}{4})2(\tau+\sigma)} + i\tilde{\alpha}^3_{-n-3/4} e^{-i(n+\frac{3}{4})2(\tau-\sigma)}) \qquad (3.5)$$

Since the orbifold has $z_3 \sim iz_3$, the boundary conditions can be such that $(\frac{\partial z_3}{\partial \sigma} \frac{\partial z_3}{\partial \tau})_{\sigma=0} = -(\frac{\partial z_3}{\partial \sigma} \frac{\partial z_3}{\partial \tau})_{\sigma=\pi}$ as they are here. Notice also that $Z_3^c = \Omega Z_3$. Trying to twist the NS and R operators acting on the vacuum by $\Omega J \alpha$ appears to lead to inconsistencies because of the extra phases of the twist operators unless one assumes that the action of Ω in these twisted sectors also reverses the GSO projection. This affect would be similar to twisting by $(-1)^{F_L}$. In order to have an invariant combination under the $\Omega\alpha$ projection, these strings must always occur in pairs, one string and its orientation reversed counterpart. If one assumes that $L_0^{string1} = L_0^{string2}$ where L_0 is the Virasoro generator, the massless spectrum is identical to the $(-1)^{F_L}\alpha$ twisted string. These two strings would result from pulling apart the left and right moving sectors of the $(-1)^{F_L}\alpha$ twisted string to create two strings. At the massive level, to produce complete agreement between the two theories probably requires modifying $(-1)^{F_L}$ to $(-1)^{F_L}\Omega_b$ or taking into account higher order corrections to the spectrum as discussed above. The spectrum of these twisted strings generally contains a massless vector and a massless hypermultiplet, but how the hypermultiplets are projected out and the gauge group becomes nonabelian in some F theory models will not be resolved here.

4. Discussion

This paper has probably raised more questions than it has resolved. We have considered orbifolds of F theory having a $\mathbf{Z_2}$ singularity on the fiber which allows for a perturbative IIB description. It is natural to wonder whether F theory orbifolds with a $\mathbf{Z_n}$ singularity, $n > 2$, that are stuck at strong coupling can be understood in any perturbative framework. The affect of discrete torsion in F theory has been seen to turn gauge symmetry into global symmetry. The significance of this result could probably be better understood. We have considered the J torsion in F theory but not understood what J corresponds to geometrically and whether tensionless strings play a role. The orientifolds

we have discussed have required a new type of twisted string that has a perturbative description. How this string fits into the D-brane framework and how a number of these strings come together to yield a nonabelian gauge symmetry is an open question. We have observed that the objects in the $\Omega \mathbf{Z_4}$ theory are "meson"-like, but one could conjecture that there are possible "baryon"-like objects formed from twisted strings. We have also found that $(-1)^{F_L}$ should be modified, at least perturbatively, to have S-duality valid for the entire spectrum. Many of the models analyzed here have been realized in seemingly different contexts such as the M theory orbifolds that are not readily converted into their F theory counterparts because of the nontrivial action on the twelfth dimension. One is therefore led to speculate about some underlying unified description.

Acknowledgments

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